Childhood Difficulties in Fraction Learning: A Critical Analysis

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There is no shortage of research investigating difficulties children experience when learning fractions (Fuchs et al., 2013; Gabriel et al., 2012; Siegler, Fazio, Bailey, & Zhou, 2013; Siegler & Pyke, 2013). Fractions are among the most difficult mathematical concepts for children to master, and many are unable to correctly identify the larger fraction in a presented fraction pair (Gabriel et al., 2012). Similar difficulties are also reported in adults, indicating that these are not problems that simply disappear over time (Gabriel, Szucs, & Content, 2013; Meert, Grégoire, & Noël, 2009; Siegler et al., 2013). Perhaps because of these difficulties, most current models of numerical development focus in the learning of whole numbers, and only mention fractions in passing to emphasize their associated learning difficulties (Leslie, Gelman, & Gallistel, 2008). One such model by Leslie and colleagues (2008) proposes that humans are born pre-programmed with a function which generates discrete integer values, thereby directing the development of whole numbers and forming the basis of arithmetic principles.

The decision to neglect fraction learning seems reasonable when contrasting fractions and whole numbers. Whole numbers are always followed by a unique successor, can be denoted with a single numeral, always increase following multiplication and decrease following division. Yet none of these are true with fractions. The only common feature shared between fractions and whole numbers is the ability to be represented on a number line (Siegler, Thompson, & Schneider, 2011).

Some researchers have gone so far as to suggest that knowledge of whole numbers impairs the learning of fractions by causing children to reflexively treat fractions as if they were whole numbers (Dehaene, 1997). Evidence of this can be seen in errors children commonly commit while performing fractional arithmetic. For example, a common error committed by children when performing fractional arithmetic is to incorrectly treat numerators and denominators as independent whole numbers (e.g., 4/5 + 3/10 = 7/15; Siegler & Pyke, 2013).

These errors may be caused by greater demands that fractions place on working memory (Fuchs et al., 2013; Siegler & Pyke, 2013). Not only must children represent two digits instead of one, but they must also inhibit a prepotent tendency to interpret each digit as an independent whole number (Siegler et al., 2013). This may be difficult for children who are still developing such executive functions (Diamond, 2002).

Generally these theories of numerical development fall short of explaining how knowledge of fractions is learned by children. A recent integrative model proposed by Siegler et al. (2011) argues that representations of numerical magnitude on a mental number line underlie both the learning of fractions and of whole numbers. This will be explored by examining the role different forms of fractional knowledge and different fraction interpretations can influence children's learning of fractional knowledge, and drawing comparisons between fraction and whole number learning. Potential implications these patterns of learning may have on current educational practices will also be discussed.

Before we can examine the support for Siegler et al.'s (2011) integrative mode, there are two main forms of mathematical knowledge that must be discussed: conceptual knowledge and procedural knowledge. Conceptual knowledge is an understanding of the principles guiding a domain- in this case, fractions (Gabriel et al., 2012). This includes processes such as the understanding of both magnitude and part-whole interpretations of fractions, which will be discussed later. Procedural knowledge is typically defined as an algorithm which can be implemented to solve a particular problem (F. Gabriel et al., 2012; S. a Hecht & Vagi, 2012). In terms of fractions, this includes every operation when performing an arithmetic calculation, such as calculating the lowest common denominator when adding or subtracting, or only inverting the second fraction when performing fraction division.

Higher levels of both conceptual and procedural knowledge have been found to be associated with greater fraction achievement (Fuchs et al., 2013; Siegler & Pyke, 2013; Siegler et al., 2011). However, most school curriculums tend to disproportionately emphasize procedural knowledge when teaching fractions to children (Fuchs et al., 2013; Gabriel et al., 2012). The reason behind this procedure-first approach is that the rote learning of arithmetic procedures forms a firm basis for students to extract and facilitate improved understanding of conceptual principles of fractions (Siegler & Pyke, 2013). While there have been studies which reported improvements in conceptual knowledge of fractions following procedural learning (Gabriel et al., 2012; Hallett et al., 2010; Hecht & Vagi, 2010), these gains are fairly minimal compared to gains in procedural knowledge (Hallett et al., 2010; Hecht & Vagi, 2010), and tend to diminish over time (Siegler et al., 2011). This gives rise to the concern that children performing these arithmetic operations are simply "on autopilot", and have no deeper understanding of what they are doing (Hallett et al., 2010; Siegler et al., 2011).

In contrast, numerous studies have reported that children with a greater conceptual understanding of fractions show greater overall fraction achievement (Fuchs et al., 2013; Hallett et al., 2010; Siegler & Pyke, 2013). Furthermore, despite the previously mentioned trend in education to focus on procedural knowledge, conceptual knowledge has been found to have a greater facilitative effect on the learning of procedural knowledge than procedural has on conceptual (Hecht & Vagi, 2010).

There are two main interpretations of fractions that compose conceptual knowledge. The interpretation most commonly used to convey the concept of fractions to American child is known as the part-whole interpretation (Fuchs et al., 2013). The part-whole interpretation is based on the intuitive observation that any fraction can be expressed as parts of a whole (Siegler et al., 2013). The part-whole interpretation is very useful and lends itself well to classroom instruction. For example, to demonstrate the concept of 3/4 to students, a teacher can simply show them a picture of a cake cut into four equal pieces with one slice missing. Yet despite these advantages, there are several limitations to the part-whole interpretation which make it insufficient on its own. One limitation is trying to teach a child improper fractions. A child seeing 6/4 may have difficulty understanding the concept of six pieces of an object divided into four parts. It is also difficult to convey the infinite divisibility of fractions, as each object is made up of discrete segments (Siegler et al., 2011).

These limitations can be overcome by employing the magnitude interpretation. This involves understanding the relationship between the numerator and the denominator, and how both define the magnitude of a fraction, rather than either number on its own (Siegler & Pyke, 2013). This is usually taught using exercises such as comparing the magnitudes of two fractions, ordering a given set of fractions, or indicating the location of a fraction on a continuous number line (Fuchs et al., 2013; F. Gabriel et al., 2012; Siegler et al., 2013, 2011).

Magnitude training has been found to improve student achievement not only on measures of magnitude understanding, but also on measures of part-whole and procedural knowledge (Fuchs et al., 2013; S. Hecht & Vagi, 2010; Siegler et al., 2013, 2011; Siegler & Pyke, 2013). Additionally, while the magnitude interpretation was found to facilitate the learning of part-whole understanding, this was not true for the reverse (Fuchs et al., 2013).

A notable example of the impact of magnitude understanding comes from a study by Fuchs and colleagues (2013). This study sought to investigate the effect of a conceptuallyfocused fraction intervention curriculum. The study consisted of two groups of 4th graders; one group consisting at-risk/low math achievers (AR), and the other group consisting of not at-risk/normal math achievers (NR). AR students participated in a 12-week long intervention where they received a specialized fraction curriculum largely centered on magnitude interpretation of fractions, with only two weeks devoted to procedural knowledge. Children were trained on tasks such as fractional magnitude comparison, ordering of fractions, and placing fractions on a 0 to 1 and 0 to 5 number line. The NR students received the typical curriculum, which focused much more heavily on procedural knowledge and the part-whole interpretation.

At pre-test, AR students were significantly lower than NR on all measures of fraction competence (Fuchs et al., 2013). However, following the 12-week intervention, AR students demonstrated significantly greater improvement on all measures. On assessments of fraction magnitude understanding, AR students scored significantly greater than NR cohorts at post-test. Interestingly, AR students also outperformed NR students on tests of fraction procedural knowledge, indicating that magnitude understanding facilitates learning of arithmetic procedures (Fuchs et al., 2013), in contradiction of the current assumptions of the curriculum (Siegler & Pyke, 2013). Other studies have reported similar improvements (Gabriel et al., 2012).

This facilitation of procedural learning can be accounted for by a number of factors associated with the magnitude tasks. One strategy children are often taught to employ when comparing fractions is chunking, where in the child decomposes a task into a series of steps, thereby reducing the load placed on the child's working memory when evaluating the magnitude of a fraction (Diamond, 2002; Fuchs et al., 2013; Siegler et al., 2011).

Representing a fraction on a number line provides a number of benefits as well. For example, a child representing a series of fractions on a 0 to 1 number line (e.g., 4/5, 3/4, 1/2, 5/8) is made aware that infinite numbers exist between zero and one (Siegler et al., 2011). This feature is crucial for understanding the infinite divisibility of fractions, yet many older children and even adolescents often fail to grasp it (Siegler et al., 2013, 2011).

However, perhaps the most interesting involving the representation of fractions on a number line comes in the enhanced ability to estimate magnitude of a fraction (Siegler et al., 2011). Practice labeling the position of a fraction has been shown to increase the accuracy and automaticity of magnitude representations in individuals (Siegler et al., 2013, 2011; Siegler & Pyke, 2013). This enhanced representation may allow children to reject obviously implausible, as well as rejecting the incorrect procedures which yielded the answer (Siegler et al., 2011). It is also possible to improve the automaticity of these judgments using strategies. For example, employing distinct marker fractions (e.g., 1/2, 1/4, etc), and comparing them with a given fraction has been shown to improve the magnitude interpretation in children (Fuchs et al., 2013).

Interestingly, the distinct marker strategy for magnitude interpretation shares similarities with models explaining the learning of whole numbers (Leslie et al., 2008). Much like the basis of whole numbers is argued to be based off of a single unit integer whose value is calibrated over time (Leslie et al., 2008), representations of these marker fractions are generated over time (Fuchs et al., 2013; Gabriel et al., 2012; Siegler & Pyke, 2013) and serve as bases of comparison for fraction magnitudes (Fuchs et al., 2013).

Therefore we see that the literature is largely in favour of the integrative model of fraction and whole number development. Contrary to current education practices, research suggests that greater emphasis should be placed on the learning of concepts rather than strict procedures (Fuchs et al., 2013; Hallett et al., 2010). In particular teachers should stress the understanding fractional magnitudes, which have the greatest facilitative effects of any form of fractional understanding (Hallett et al., 2010; Siegler et al., 2013, 2011; Siegler & Pyke, 2013).

These facilitative transfer effects may be cause by magnitude representation on a mental number line, which may underlie general numerical and arithmetic processes (Miller & Cohen, 2001). This also seems logical given findings in neuroscience which indicate that numerical magnitudes share a common neural representation in the intraparietal sulcus (Ansari, 2008).

Some objections may be raised to this curriculum shift in regards to children’s part-whole understanding. While magnitude understanding facilitated part-whole learning, children educated using magnitude-centric lessons still underperformed on part-whole measures compared to children educated using the standard curriculum (Fuchs et al., 2013). However, this may be amenable by a simple change in phrasing. A study by Miura and colleagues (Miura, Okamoto, Vlahovic-Stetic, Kim, & Han, 1999) found that simply educating children using characteristics of East Asian mathematical language which transparently convey the part-whole/numerator-denominator relationship (e.g., 1/4 would be “of four parts, one”) allowed US 2nd graders to perform match the typically higher-scoring Korean students on part-whole measures.

Future research should continue explore the facilitative and inhibitory relationships between these forms of fractional knowledge, as well as demonstrate how they may change over time. Studies should also explore how these relationships may vary between individuals, and how these variations can be accounts for in the curriculum.

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